Normal Percentiles

Lecture 21
Section 6.3.1

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Outline

1. z-Scores
2. Standard Normal Percentiles
3. Normal percentiles
4. Assignment

Review Quiz

Example (Review Quiz)

1. Men's IQ scores have a greater standard deviation than women's IQ scores. Therefore, (choose as many as apply)
   (a) The men's distribution is to the right of the women's.
   (b) The men's distribution is taller than the women's.
   (c) The men's distribution is wider than the women's.
   (d) The men's distribution is narrower than the women's.

Example (Review Quiz)

2. Furthermore, the means of the men's and women's IQ scores are the same. Therefore, (choose as many as apply)
   (a) More highly intelligent people are men.
   (b) More low-intelligence people are women.
   (c) More low-intelligence people are men.
   (d) Highly intelligent people are evenly divided between men and women.

Review Quiz Answers

Example (Review Quiz Answers)

1. (a)
2. (a), (c)

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z-Scores

Definition (z-score (or standard score))

The z-score (or standard score) of a member of a sample or population is the number of standard deviations between that value and the mean.

- Compute the z-score of x as

$$z = \frac{x - \overline{x}}{s}.$$

- Equivalently

$$x = \overline{x} + zs.$$

Example

Example (z-scores)

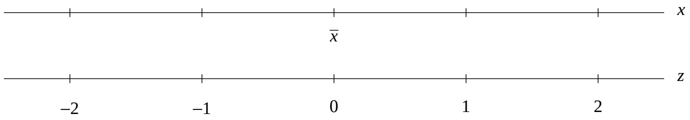
- Let X be the height of adult U.S. males, with distribution $N(69.5, 2.9)$.
- What proportion of values of X are between 65 and 72?
- Compute z-score of 65 is

$$\frac{65 - 69.5}{2.9} = \frac{-4.5}{2.9} = -1.55.$$

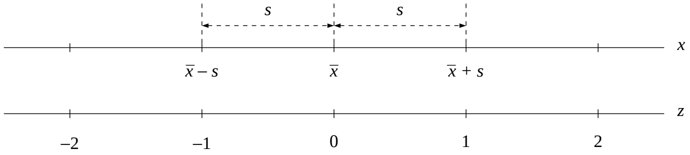
- Compute z-score of 72 is

$$\frac{72 - 69.5}{2.9} = \frac{2.5}{2.9} = 0.86.$$

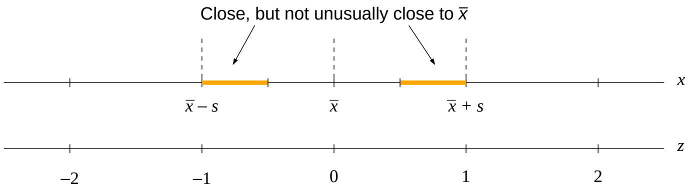
Interpreting the Standard Deviation



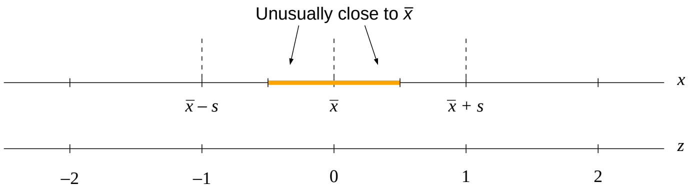
Interpreting the Standard Deviation



Interpreting the Standard Deviation



Interpreting the Standard Deviation



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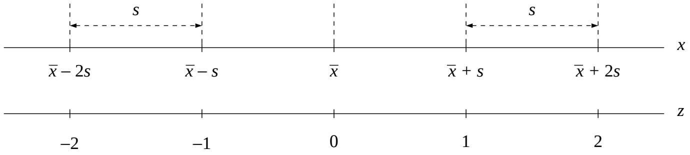

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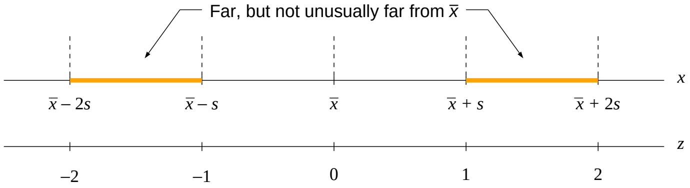

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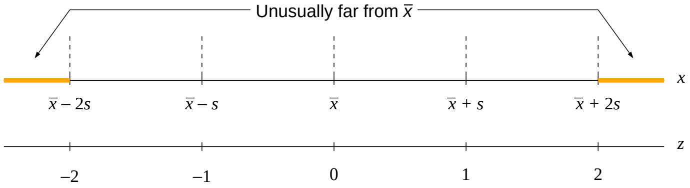

Example

Example (z-scores)

- Now find the area between -1.55 and 0.86 under the *standard* normal curve.

$$\texttt{normalcdf(-1.55,0.86)} = 0.7445.$$

- Therefore, 74.45% of the values of X are between 65 and 72.

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Two Problems

- The problem so far:

  Given the values of the variable $\longrightarrow$ Find the proportion.

- The next problem:

  Given the proportion $\longrightarrow$ Find the values of the variable.

- The second problem is the reverse of the first problem.

Two Problems

- Find a normal probability:

  Given the value of z $\rightarrow$ Find the area to the left of z.

- Find a normal percentile:

  Given the area to the left of z $\rightarrow$ Find the value of z.

- The second problem is the reverse of the first problem.

Standard Normal Percentiles

Example (Standard normal percentile)

- What is the 90th percentile of z?
- That is, find the value of z such that the area to the left is 0.9000.
- On the TI-83, use the `invNorm` function.

TI-83 - Standard Normal Percentiles

TI-83 Standard normal percentiles

- Press `2nd DISTR`.
- Select `invNorm` (Item #3).
- Enter the percentile rank as a decimal (i.e., the area).
- Press `ENTER`. The percentile appears in the display.

TI-83 - Standard Normal Percentiles

Practice

- Use the TI-83 to find the following percentiles.
  - Find the 90^{th} percentile of z.
  - Find the 1^{st} percentile of z.
  - The value of z that cuts off the top 20%.
  - Find Q_1 and Q_3 of z (middle 50%).
  - The values of z that determine the middle 30%.

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Example

Example (Normal percentiles)

- Assume that adult U.S. male heights are $N(69.5, 2.9)$.
- What is the 90^{th} percentile of male heights?
- The 90^{th} percentile of z is 1.282.
- Therefore, the 90^{th} percentile for male heights is

$$69.5 + (1.282)(2.9) = 73.2.$$

- That is, 90% of male heights are below 73.2.

TI-83 - Normal Percentiles

TI-83 Normal percentiles

- Use `invNorm` to find the standard normal percentile and use the equation $x = \mu + z\sigma$ (as in the previous example).
- Or, use `invNorm` and specify the percentile rank, μ, and σ.

- For example, in the previous problem,

$$\texttt{invNorm(0.90,69.5,2.9)} = 73.2.$$

TI-83 - Normal Percentiles

Practice

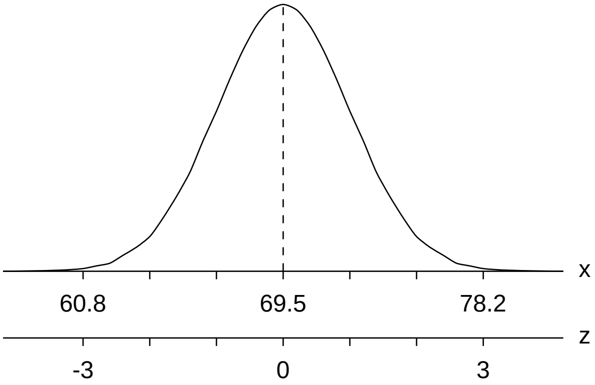
- Find the 80^{th} percentile of male heights.
- Find the first and third quartiles of male heights.

The Standard Normal Curve

- If a variable X has a normal distribution, then the z-scores of X have a standard normal distribution.
- That is, if X is $N(\mu, \sigma)$, then $\dfrac{X - \mu}{\sigma}$ is $N(0, 1)$.
- In other words,

$$Z = \frac{X - \mu}{\sigma}.$$

The Standard Normal Curve



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Assignment

Homework

- Read Section 6.3.1, pages 370 - 376.
- Let's Do It! 6.7, 6.8.
- Exercises 14, 17, 19, 22 - 27, page 376.